

Mathematical induction's fixpoint

Let r be a left-founded relation, i.e. let

$$(0) \quad \langle \forall x :: [x \Rightarrow r; x] \Rightarrow [x \Rightarrow \text{false}] \rangle \quad ;$$

an appeal to what is called "mathematical induction" means that a proof obligation of the form

$$(1) \quad [x \Rightarrow \text{false}]$$

is replaced by the formally weaker obligation

$$(2) \quad [x \Rightarrow r; x] \quad \text{or} \quad [x \wedge \neg(r; x) \Rightarrow \text{false}].$$

Proving (2) by mathematical induction leads to the (subsequently simplified) obligation

$$\begin{aligned} (3) \quad & [x \wedge \neg(r; x) \Rightarrow r; (x \wedge \neg(r; x))] \\ \equiv & \quad \{\text{shunting}\} \\ & [x \Rightarrow r; x \vee r; (x \wedge \neg(r; x))] \\ \equiv & \quad \{; \text{monotonic in 2nd argument}\} \\ & [x \Rightarrow r; x] \end{aligned}$$

i.e. (2)! The decision to prove by mathematical induction is idempotent. With thanks to the ETAC.

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