

The equation $x: [x]$

For equality between "predicates" x and y we introduce the notation $[x \equiv y]$. For f a function of type $\text{predicate} \rightarrow \text{predicate}$ Leibniz's Principle is accordingly expressed as

$$[x \equiv y] \Rightarrow [f.x \equiv f.y]$$

We now dissect this notation into

- (i) the square brackets $[]$, which denote a function of type $\text{predicate} \rightarrow \{\text{true}, \text{false}\}$
- (ii) the infix operator $\equiv$ which is of type $\text{predicate}^2 \rightarrow \text{predicate}$ and is postulated to be associative and symmetric.

Parsing the relation $[(x \equiv y) \equiv (y \equiv x)]$ that expresses the symmetry as $[x \equiv ((y \equiv y) \equiv x)]$ and $[(x \equiv (y \equiv y)) \equiv x]$, we see that $y \equiv y$ is both left and right identity element of $\equiv$. Since for an infix operator that has a left identity element and a right identity element, these are equal and unique, we can name the unique identity element of $\equiv$, say by TRUE:

$$[x \equiv \text{TRUE} \equiv x]$$

This immediately leads to

$$[x] \Rightarrow [x \equiv \text{TRUE}]$$

i.e. equation $x: [x]$ has a unique solution which equals the identity element of $\equiv$ (and, hence, $[x]$ is false for all other values of x).

Remark. It is usual to write TRUE in lower case. (End of Remark.)

The above has been written for David Gries.

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