

A theorem about "factors" perhaps worth recording

Let $\backslash$ be defined by

$$(0) \quad [x; y \Rightarrow z] \equiv [y \Rightarrow x \backslash z] \quad ;$$

(0) defines $x \backslash z$ as the weakest solution of
 $y: [x; y \Rightarrow z]$

and yields with instantiations $y := x \backslash z$ and
 $z := x; y$ respectively

$$(1) \quad [x; x \backslash z \Rightarrow z]$$

$$(2) \quad [y \Rightarrow x \backslash (x; y)]$$

(We have given $\backslash$ a higher binding power than $;$.)

About the transpose $\sim$ (prefix) - which others call the converse $\circ$ (postfix) - I shall use the Dedekind Law

$$(3) \quad [x; y \wedge z \Rightarrow x; (y \wedge \sim x; z)] \quad .$$

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We shall now prove

$$(4) \quad [p; q \wedge \sim p \backslash r \Rightarrow p; r]$$

To this end we observe for any p, q, r

$$\Rightarrow \quad p; q \wedge \sim p \backslash r$$

{ (3) with $x, y, z := p, q, \sim p \backslash r$ }

$$\begin{aligned} & p; (q \wedge \sim p; \sim p \setminus r) \\ \Rightarrow & \{ \text{monotonicties, (1) with } x, z := \sim p, r \} \\ & p; r \end{aligned}$$

I used (4) to prove the theorem of section 2.3 of "A Graphical Calculus" by Sharon Curtis and Gavin Lowe, Oxford University Computing Laboratory, Parks Road, Oxford, OX1 3QD; the formulation of (4) was triggered by their note.

An alternative formulation of (4) that incorporates the antimonotonicity of $\setminus$ in its left argument is

$$(5) \quad [\sim p \Rightarrow s] \Rightarrow [p; q \wedge s \setminus r \Rightarrow p; r] .$$

I think the theorem is worth recording though not worth remembering.

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