

Introducing a course on mathematical methodology

For a number of very different reasons, which will become clear in the course of this introduction, I cannot just begin with the subject matter of the course. Some explanations and warnings are needed first, because without them you probably would not know how to listen to me.

The warnings are mostly about me. Since I am in Austin, I have learned that I seem to address to types of students: on the one hand there are the students that have greatly appreciated my lectures as an enriching experience because they were so totally different from the others they were exposed to, on the other hand there are the students that blamed me for deviating from what they were used to.

Well, I don't intend to try to adjust my lecturing style to the expectations of the latter category. Firstly, I know that, never having been exposed to the American educational system, I would be unable to imitate my average American colleague. Secondly, I would be forsaking my educational duty if I tried: I have not been invited to come to UT so that I could offer what any knowledgeable experienced American colleague can offer better, I have been invited to complement the system and to represent what it tends to neglect.

To put it bluntly, a major part of my educational duty is to be an un-American activity become flesh.

More concretely, whenever in this course a clash of cultures surfaces, we should remember that exposure to a different scientific and educational tradition had been planned, right at the start, as an essential ingredient of the course.

It is our combined duty, but primarily mine, to prevent this clash of cultures from becoming destructive. Your contribution to this effort is, for the moment, to listen as open-mindedly and as carefully as possible; my contribution will be to prepare you, as carefully as I can, for the differences with which you will be confronted.

I grew up in a society where the transition from secondary school to university was a very sharp one, underlined by all the accompanying linguistic symptoms. The university had professors, students, and lecture rooms, the school had teachers, pupils, and class-rooms. At school we were addressed by our first name and in the familiar form, at the university by our family name and in the polite form. In short: abruptly we were treated as grown-up. We were addressed as "Ladies and gentlemen" and expected to behave accordingly.

As we had just witnessed from uncomfortably close quarters the near collapse of Western civilization, the cultural and scientific enterprise as embodied by the universities was taken very seriously, notwithstanding our awareness that not all of the academic community lived up to those high standards - as illustrated by the definition of a university professor as someone who casts false pearls before real swine - . But the standards

very much alive. It was our task to mature as cultured, competent scholars and scientists and no one expected us to prepare ourselves for a profession: that was done at trade schools and a university was not a trade school. (It would never have entered our minds to describe a professor as "someone who hasn't made it in industry"; the relation was probably the inverse in that industry had to make do with the left-overs of the academic world.) We had, in general, great respect for our professors and, conversely, they had great respect for quite a few of us. Talking down to one's students was something that was definitely "not done", and the rare professor that mistakenly did it was despised for it.

Certainly during my earlier years the idea of hand-outs would have been preposterous: we could make our own notes! When, in my later years, the professor of theoretical physics - he had come from the University of Utrecht - handed out a syllabus, we felt deeply offended. We experienced his excerpt as an obnoxious barrier between us and the original works from which it had been extracted. We accused him of contempt of audience and eventually he went to the University of Amsterdam.

Such were the formative years from which date the pan-academic prejudices that now parade as my well-considered opinions. They have profound consequences for the way in which I propose to give this course. Let me give you a few warnings.

- I shall give an occasional hand-out - after all we are now more than a third of a century later-. But in principle

you are required to make your own notes and to work them out between lectures. This is for you the only way of beginning to acquire the necessary penmanship and of discovering in time that you failed to understand what seemed so clear and simple when I explained it. This is not a vain warning: for decades I have been accused of telling such smooth stories that half my audience fails to see most of the potential difficulties.

- Remember: you are grown-up. That means that it is not my responsibility to teach you something; my only responsibility is to give you the opportunity to learn, preferably a lot and stuff you cannot learn from someone else. I may give some exercises at which you may try your hand, I shall not give homework that must be handed in to be graded. I am not forcing you to listen to me, it is your responsibility to try to learn as much as possible. Remember: you are grown-up.

- For a successful course, mutual respect is essential, and this mutual respect has to be earned. I, from my side, intend to do that by not using the poison of the educational process known as "the overhead projector", by trying to perfect my chalkmanship, by addressing you in carefully considered and grammatically complete sentences, and by drawing your attention to a host of essential particulars that otherwise would have escaped your attention. You, from your side, have to earn my respect as well. That goes beyond laughing about my jokes - just to show that you have noticed them - and correcting me when I have omitted a closing parenthesis. Your duty is such an active and critical participation that what unavoidably starts as a monologue develops into a meaningful dialogue.

Your rôle is not one of sponges, but one of whetstones; only then the spark of intellectual excitement can continue to jump over.

- A final warning about my personal style. I am used to expressing myself tersely, a tendency that can only have been strengthened by the circumstance that I have to address you in what for me is a foreign language. I am not one of those so-called gifted speakers that can just turn their mouths loose; I am one of those unfortunate people that have to think about what they are saying. As a result I speak slowly, so slowly that I can pause in mid-sentence. But, for Heaven's sake, don't be deluded: in all probability you have to listen twice as fast as you are used to. I grew up in a culture in which, if something had been said once, that should suffice: you had taken note, hadn't you? Let me, by way of exception, illustrate this point. I said "I am used to express myself tersely" and left it at that because the adjective "tersely" conveys the message accurately; I abstained from explaining the term for greater emphasis by adding "that is, I tend to use as few words as needed and tend not to repeat myself". [To this very day, idiot software managers measure "programmer productivity" in terms of "lines of code produced", whereas the notion of "lines of code spent" is much more appropriate. With words, it is in the same way.] In short, at the end of a lecture you should be at least as tired as I.

So much for the style in which this course will be presented. Let me now turn to the nature of the material I intend to cover.

The idea of giving this course has been born out of dissatisfaction with the ways in which mathematics has usually been taught since I was a student. The courses I followed consisted of facts, facts, and more facts: the triple (definition, theorem, proof) repeated ad infinitum. Very useful if you wished to apply or extend the theory presented and totally useless otherwise. Later I saw a different type of mathematics course emerge: tricks, tricks, and more tricks, recipes for solving very special types of problems and endless exercises so as to make their execution a routine job. Useful perhaps when that routine job is what you are heading for, but, again, totally useless otherwise. Furthermore, particularly in countries where rhetoric is confused with effective use of language, we have seen courses that are cult of pure form: the student is only taught a pompous language and mathematics is reduced to mannerisms.

The student that, like the wild animal being prepared for its tricks in the circus called "life", expects only training as sketched above, will be severely disappointed: by his standards he will learn nothing.

Let me quote, by way of contrast and for inspiration, what G.R.Elton wrote about the teaching of history:

"Three or four years spent at a university cannot teach a man to know history; they cannot train him as a politician or publicist or publisher; they can at best begin to lay some foundations for a view of the world and (universities being what they are) are likely to lay foundations

which, as later experience shows, need to be broken up. None of this invites blame: the impossible need not be attempted. But if those years do not produce an effective conditioning of the reasoning mind, if they do not teach a man to think better than otherwise he would have done, they may justly be condemned as a waste of time."

Well, I would be grieved to hear if your years at UT could, in Elton's words, "justly be condemned as a waste of time". In the name of the effective conditioning of the reasoning mind, in order to teach you to think better than otherwise you would have done, this course will be about the doing of mathematics. It is my purpose to bring into the public domain what in the traditional mathematics course is left implicit.

It is not my purpose to turn, in a one-semester course, all of you into as many excellent mathematicians. Remember: the impossible need not be attempted. But I can and shall try to give you the opportunity of becoming a better mathematician than otherwise you would have been and I propose to do so by exposing at least the major components of the mathematical craft and by warning you explicitly for some of the more common mistakes that the experienced competent mathematician avoids out of habit.

How to raise such a course above the level of motherhood statements - the level that is character-

istic of the management courses sold by the training industry - has been told to us by William Blake (1757-1827)

"He who would do good to another must do it in minute particulars

General Good is the plea of the scoundrel,
hypocrite and flatterer

For Art and Science cannot exist but in minutely organized particulars "

The moral of this quotation is that, in principle, no detail will be too small for our attention. I also like this quotation for the way in which it mentions Art and Science in the same breath, for, indeed, the analogy between the performing artist and the productive scientist is not so far-fetched at all. Among the very basic things the young pianist has to learn is how to take care of his finger nails; we shall encounter similar care, of our mathematical finger nails, so to speak. Also, the performing artist must become very familiar with the potential and the limitations of his instruments; so must we with our instruments such as pen, paper, and the printed page.

The analogy goes further. Just as a professor at a conservatory represents a musical style (to the extent that it is often possible to identify the master by listening to his pupils), I represent a mathematical style. It is up to you to decide later to what degree to adopt and to improve it. One thing, however, you are not allowed to do, viz. to reject it offhand for the sole reason that it does not reflect the way

of doing mathematics you are used to. Of course it doesn't! That is precisely why you are here. This whole course is no more and no less than an invitation to take the experiment of changing some of your reasoning habits and adopting some new modes of expression. As you take the experiment you will notice that it is not acquiring the new habits that presents the greatest problem, for that is getting rid of the old ones. Perfecting oneself is as much unlearning as it is learning.

This is obvious. Yet it needs to be stressed because stupid educationists have invented for the educational process the profoundly inadequate term "knowledge transfer", suggesting a unidirectional stream of knowledge towards an accumulating recipient, who becomes monotonically more knowledgeable. I consider it a caricature at best. The trouble is that it may have struck in some of your minds, thereby creating an extra barrier for the unlearning process. It makes some people totally unprepared for the recognition that among what they have acquired so far is possibly a lot of junk that is more a burden than an asset and that they should be happier without. One cannot be cautious enough with the choice of one's past.

Austin, June 1986

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